A Benchmark TEAM Problem for Multiobjective Pareto Optimization of Electromagnetic Devices

Paolo Di Barba[1], Maria Evelina Mognaschi[1], *Member, IEEE*, Song Xiao[2], David A. Lowther[3], *Fellow, IEEE*, and Jan K. Sykulski[4], *Fellow, IEEE*

[1]Department of Electrical, Computer and Biomedical Eng, University of Pavia, Italy, {paolo.dibarba,eve.mognaschi}@unipv.it
[2]Southwest Jiaotong University, Chengdu, China, xiaosong@home.swjtu.edu.cn
[3]McGill University, Montreal, Canada, david.lowther@mcgill.ca
[4]Electronics and Computer Science, University of Southampton, Southampton, UK, jks@soton.ac.uk

The paper proposes a new benchmark for multiobjective optimization. A solution is furnished which has enabled an extensive search and reliable estimation of the shape of the Pareto front. Field uniformity and sensitivity are considered in the context of robust design. It is argued that the benchmark will provide a challenging target for new algorithms, especially those involving numerical modelling using finite element codes where the number of objective function calls needs to be minimized for practical designs.

***Index Terms*—Design optimization, multi-objective optimization, sensitivity analysis, Pareto front.**

I. Introduction

TEAM problems are well established and available from the website of the International Compumag Society [1]. In the area of optimization of electromagnetics devices there are two particular benchmarks known as No 22 SMES Optimization Benchmark and No 25 Optimization of Die Press Model. Both have been used extensively for testing new single-objective optimization algorithms. For some time there has been a need to create a benchmark which could ultimately be used for multi-objective problems, with particular emphasis on Pareto optimization. In this paper we put forward a simple – but often important in practical applications – model for assessing the quality of a magnetic field produced by a distributed winding. The field uniformity is usually of prime interest but sensitivity is also an important issue, especially in the context of robust design. The available design space has been extensively searched to yield a reliable non-dominated solution for further consideration in a Pareto sense. Optimization algorithms may therefore be tested against the proposed benchmark

II. Electromagnet Optimal Design: Model Specification

Suitably arranged current-carrying coils can be utilized to synthesize a magnetic field with a desired distribution. In biomedical engineering, for instance, there are many relevant applications: a uniform magnetic field is the background of nuclear magnetic resonance spectroscopy, while a linear profile of the field is a prerequisite for magnetic resonance imaging. Moreover, in magneto-fluid hyperthermia (MFH) the field uniformity helps to achieve a uniform distribution of heat generated in the nano-particle fluid previously injected in the target region, *e.g.* a tumor mass under treatment [2]. Thus the idea behind this benchmark problem has been inspired by important practical applications.

Consider an air-cored multi-turn winding. A non-trivial inverse problem is the synthesis of the magnetic field along the solenoid axis. This problem can be formulated as follows: given the current density J, find the distribution of turn radii $r(z)$, $-d \le z \le d$ that yields the prescribed flux density $B_0(z)$ in a one-dimensional sub-region $-c \le z \le c$ along the solenoid axis. After integrating the simple equation which describes the flux density due to an elementary turn with an internal radius r_i, an external radius r_s, and carrying a current Jdz, the following expression for the flux density at point z holds

$$B(z)=\frac{\mu_0}{2}\int_{-d}^{d}\int_{r_i}^{r_s}\frac{Jr^2(\xi)\,dr\,d\xi}{\sqrt{\left[r^2(\xi)+(z-\xi)^2\right]^3}} \qquad (1)$$

If $B(z) = B_0(z)$, $-c \le z \le c$, is the prescribed function and $r(\xi)$, $-d \le \xi \le d$, is the unknown function, then (1) implies that the field distribution which is required along the winding axis can be synthesized by imposing a suitable distribution of the turn radii. To focus the attention, a small-size solenoid, as used for *in vitro* experiments of MFH, is considered here, with the following values of the main parameters: d = 15 mm, r_i = 9 mm, r_s = 10 mm, c = 5 mm; the height of each turn is h = 1.5 mm. The winding is composed of n_t = 20 series-connected turns, thus, assuming a symmetric distribution, ten unknown radii (design variables) are to be identified. In turn, the flux density is prescribed in n_p = 41 sample points, evenly spaced along the solenoid axis.

It is important to note at this stage that in a classical design problem of air-cored multi-turn inductors, where B is given, the current distribution J is unknown and the radii distribution r is assigned, (1) becomes the Fredholm's integral equation of the first kind which can be solved using standard techniques. In fact the relationship between B and J in (1) is linear and it is straightforward to define the kernel linking B to J. In contrast, the solution to the proposed design problem – when J is assigned and r is unknown – does not lead to a Fredholm's integral equation of the first kind and, therefore, it is substantially more complicated. Moreover, the relationship between B and r in (1) is non-linear and the kernel linking B to r cannot be defined.

We also propose that an additional design criterion should be introduced considering electromagnet sensitivity against small errors in placing or shaping the turns. The objective then becomes to reduce the sensitivity as much as possible, without upsetting the requirement of a particular field distribution within a certain tolerance, thus making the design more robust.

III. Uniformity versus Sensitivity

Following the argument of Section II, the electromagnet design problem can therefore be reformulated as a bi-objective optimisation problem: *find the family of r-distributions that minimise the discrepancy – or field residual – between the prescribed* B_0 *and the actual induction field B*

$$f_1(r) = \sup_{q=1,n_p} \left| B\left(z_q, r(\xi_\ell)\right) - B_0\left(z_q\right) \right| , \ \ell = 1, n_t \qquad (2)$$

and simultaneously minimise the following sensitivity function

$$f_2(r) = \sup_{\ell=1,n_t} \left[\left\| B^+ - B\left(r(\xi_\ell)\right) \right\| + \left\| B\left(r(\xi_\ell)\right) - B^- \right\| \right] \qquad (3)$$

where B^+ and B^- are the flux density values computed after an expansion, or a contraction, of all radii with respect to the unperturbed configuration, respectively:

$$B^+ = B\left(r(\xi_\ell + \Delta\xi)\right) \ , \ B^- = B\left(r(\xi_\ell - \Delta\xi)\right) \ , \ \ell = 1, n_t \qquad (4)$$

where the amount of expansion/contraction is constant and equal to $\Delta\xi = 0.05$ mm.

IV. Results and Discussion

The analysis problem – *i.e.* given *J* and *r* distributions, find the flux density field *B* – was solved using a finite-element axisymmetric model [3]. Fig. 1 shows the approximated Pareto front trading off uniformity and sensitivity of the air-cored electromagnet. In (2) a constant field of value B_0 = 5mT along the controlled region [–5, 5] mm of the winding axis was assumed. The relevant set of nine solutions was obtained by means of the goal-attainment method with variable weights [4]. Table I shows the detail of the Pareto optimal solutions.

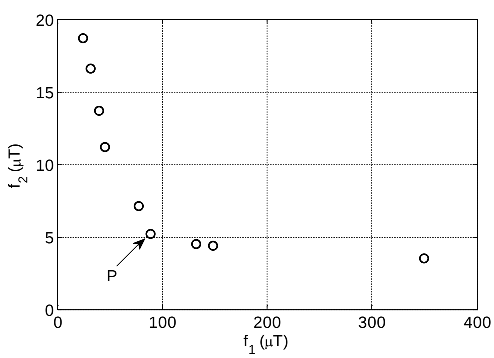

Fig. 1. Pareto front of the problem defined by (2) and (3).

TABLE I
Coordinates of the Pareto-optimal solutions in both design space and objective space (R_k , k=1,10 in mm; f_1 and f_2 in μT)

weight	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	f_1	f_2
[0.1,0.9]	10.54	9.24	7.51	5.07	3.12	3.60	5.43	3.87	4.65	4.24	24.4	18.7
[0.2,0.8]	9.67	8.28	6.75	5.03	2.48	3.91	4.11	3.74	4.23	4.10	31.6	16.6
[0.3,0.7]	12.10	11.21	9.64	9.19	10.28	2.51	6.58	3.76	7.17	4.74	39.7	13.7
[0.4,0.6]	9.27	8.98	7.62	6.81	8.46	2.54	6.53	3.69	5.44	4.38	45.3	11.2
[0.5,0.5]	7.81	6.49	4.53	3.71	2.14	3.42	3.27	3.31	3.30	3.35	78.0	7.1
[0.6,0.4]	9.57	7.84	6.72	3.85	3.31	3.44	4.54	3.86	4.20	4.06	89.0	5.2
[0.7,0.3]	4.59	4.00	3.47	2.76	1.66	2.12	2.34	2.29	2.33	2.33	132.3	4.5
[0.8,0.2]	4.53	4.07	3.38	2.80	1.86	2.24	2.42	2.39	2.42	2.42	148.2	4.4
[0.9,0.1]	4.15	3.73	3.34	2.44	2.28	2.27	2.38	2.47	2.45	2.45	350.1	3.5

The minimum-norm solution, *i.e.* the Pareto-optimal solution closest to the utopia point (point P in Fig. 1), is shown in Fig. 2.

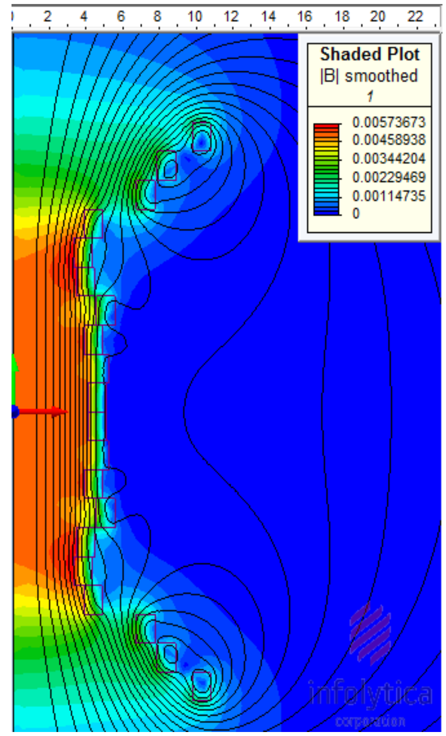

Fig. 2. Minimum-norm Pareto-optimal solution.

V. Conclusions

A TEAM benchmark problem has been proposed, and a feasible solution derived, to establish a Pareto front. The objective functions have been specified as field uniformity and sensitivity resulting in a particular shape of the Pareto front presenting a challenge to optimizers. The design space has been thoroughly searched to provide a definite location of the non-dominated solution. Further variants of the benchmark problem will be discussed in the full version and various methods of multi-objective optimization will be used, *e.g.* the well-known NSGA-II, the new biogeography based algorithm [5], the multi-objective wind-driven [6] optimization and kriging assisted algorithms [7]. Moreover, the proposed benchmark shows that even in the case of quite simple geometries – and straightforward analysis tasks – complicated inverse problems may arise.

References

[1] http://www.compumag.org/jsite/team.html

[2] R. Bertani, F. Ceretta, P. Di Barba, M. Forzan, F. Dughiero, R. Michelin, P. Sgarbossa, E. Sieni, and F. Spizzo, “Optimal inductor design for nanofluid heating characterisation,” *Engineering Computation*, vol. 32, no. 7, pp.1870–1892, 2015.

[3] http://www.infolytica.com

[4] P. Di Barba, *Multiobjective Shape Design in Electricity and Magnetism*, Springer, 2010.

[5] P. Di Barba, F. Dughiero, M.E. Mognaschi, A. Savini, and S. Wiak, “Biogeography-Inspired Multiobjective Optimization and MEMS Design,” *IEEE Transactions on Magnetics*, vol. 52, no. 3, 2016.

[6] P. Di Barba, “Multi-objective Wind Driven Optimisation and Magnet Design,” *Electronics Letters*, vol. 52, no. 14, pp. 1216-1218, 2016.

[7] Li Yinjiang, Xiao Song, M. Rotaru, and J. K. Sykulski, “A dual kriging approach with improved points selection algorithm for memory efficient surrogate optimization in electromagnetics,” *IEEE Transactions on Magnetics*, vol. 52, no. 3, 2015.